Chapter 7

Problems: 2, 9, 15, 23, 34, 38, 39, 41, 47, 92 and 98
Think about: 19, 71, 79 and 107

2 • Two stones are simultaneously thrown with the same initial speed from the roof of a building. One stone is thrown at an angle of 30° above the horizontal, the other is thrown horizontally. (Neglect effects due to air resistance.) Which statement below is true?
(*a*) The stones strike the ground at the same time and with equal speeds.
(*b*) The stones strike the ground at the same time with different speeds.
(*c*) The stones strike the ground at different times with equal speeds.
(*d*) The stones strike the ground at different times with different speeds.

Determine the Concept Choose the zero of gravitational potential energy to be at ground level. The two stones have the same initial energy because they are thrown from the same height with the same initial speeds. Therefore, they will have the same total energy at all times during their fall. When they strike the ground, their gravitational potential energies will be zero and their kinetic energies will be equal. Thus, their speeds at impact will be equal. The stone that is thrown at an angle of 30° above the horizontal has a longer flight time due to its initial upward velocity and so they do not strike the ground at the same time. $\boxed{(c)}$ is correct.

9 • Assume that, when the brakes are applied, a constant frictional force is exerted on the wheels of a car by the road. If that is so, then which of the following are necessarily true? (*a*) The distance the car travels before coming to rest is proportional to the speed of the car just as the brakes are first applied, (*b*) the car's kinetic energy diminishes at a constant rate, (*c*) the kinetic energy of the car is inversely proportional to the time that has elapsed since the application of the brakes, (*d*) none of the above.

Picture the Problem Because the constant friction force is responsible for a constant acceleration, we can apply the constant-acceleration equations to the analysis of these statements. We can also apply the work-energy theorem with friction to obtain expressions for the kinetic energy of the car and the rate at which it is changing. Choose the system to include the earth and car and assume that the car is moving on a horizontal surface so that $\Delta U = 0$.

(*a*) A constant frictional force causes a constant acceleration. The stopping distance of the car is related to its speed before the brakes were applied through a constant-acceleration equation.

$$v^2 = v_0^2 + 2a\Delta s$$

or, because $v = 0$,

$$0 = v_0^2 + 2a\Delta s \Rightarrow \Delta s = \frac{-v_0^2}{2a}$$

where $a < 0$.

Thus, $\Delta s \propto v_0^2$: Statement (*a*) is *false*.

(*b*) Apply the work-energy theorem with friction to obtain:

$$\Delta K = -W_\text{f} = -\mu_\text{k} mg\Delta s$$

Express the rate at which K is dissipated:

$$\frac{\Delta K}{\Delta t} = -\mu_\text{k} mg\frac{\Delta s}{\Delta t}$$

Thus, $\frac{\Delta K}{\Delta t} \propto v$ and therefore not constant. Statement (*b*) is *false*.

(*c*) In Part (*b*) we saw that: $K \propto \Delta s$

Because $\Delta s \propto \Delta t$ and $K \propto \Delta t$: Statement (*c*) is *false.*

Because none of the above are correct: $\boxed{(\boldsymbol{d})}$ is correct.

15 •• **[SSM]** Assume that your maximum metabolic rate (the maximum rate at which your body uses its chemical energy) is 1500 W (about 2.7 hp). Assuming a 40 percent efficiency for the conversion of chemical energy into mechanical energy, estimate the following: (*a*) the shortest time you could run up four flights of stairs if each flight is 3.5 m high, (*b*) the shortest time you could climb the Empire State Building (102 stories high) using your Part (*a*) result. Comment on the feasibility of you actually achieving Part (*b*) result.

Picture the Problem The rate at which you expend energy, that is do work, is defined as *power* and is the ratio of the work done to the time required to do the work.

(*a*) Relate the rate at which you can expend energy to the work done in running up the four flights of stairs:

$$\varepsilon P = \frac{\Delta W}{\Delta t} \Rightarrow \Delta t = \frac{\Delta W}{\varepsilon P}$$

where *e* is the efficiency for the conversion of chemical energy into mechanical energy.

The work you do in climbing the stairs increases your gravitational potential energy:

$$\Delta W = mgh$$

Substitute for ΔW to obtain:

$$\Delta t = \frac{mgh}{\varepsilon P} \quad (1)$$

Assuming that your mass is 70 kg, substitute numerical values in equation (1) and evaluate Δt:

$$\Delta t = \frac{(70\text{ kg})(9.81\text{ m/s}^2)(4\times 3.5\text{m})}{(0.40)(1500\text{W})} \approx \boxed{16\text{s}}$$

(*b*) Substituting numerical values in equation (1) yields:

$$\Delta t = \frac{(70\text{ kg})(9.81\text{ m/s}^2)(102\times 3.5\text{m})}{(0.40)(1500\text{W})} = 409\text{ s} \approx \boxed{6.8\text{ min}}$$

The time of about 6.8 min is clearly not reasonable. The fallacy is that you cannot do work at the given rate of 250 W for more than very short intervals of time.

19 •• Hydroelectric power plants convert gravitational potential energy into more useful forms by flowing water downhill through a turbine system to generate electric energy. The Hoover Dam on the Colorado River is 211 m high and generates 4×10^{-9} kW·h/y. At what rate (in L/s) must water be flowing through the turbines to generate this power? The density of water is 1.00 kg/L. Assume a total efficiency of 90.0 percent in converting the water's potential energy into electrical energy.

Picture the Problem We can relate the energy available from the water in terms of its mass, the vertical distance it has fallen, and the efficiency of the process. Differentiation of this expression with respect to time will yield the rate at which water must pass through its turbines to generate Hoover Dam's annual energy output.

Assuming a total efficiency ε, use the expression for the gravitational potential energy near the earth's surface to express the energy available from the water when it has fallen a distance h:

$$\boldsymbol{E = \varepsilon mgh}$$

Differentiate this expression with respect to time to obtain:

$$P = \frac{d}{dt}[\varepsilon mgh] = \varepsilon gh\frac{dm}{dt} = \varepsilon\rho gh\frac{dV}{dt}$$

Solving for dV/dt yields:

$$\frac{dV}{dt} = \frac{P}{\varepsilon\rho gh} \quad (1)$$

Using its definition, relate the dam's annual power output to the energy produced:

$$P = \frac{\Delta E}{\Delta t}$$

Substituting for P in equation (1) yields:

$$\frac{dV}{dt} = \frac{\Delta E}{\varepsilon \rho g h \Delta t}$$

Substitute numerical values and evaluate dV/dt:

$$\frac{dV}{dt} = \frac{4.00\times10^{9}\ \text{kW}\cdot\text{h}}{(0.90)(1.00\,\text{kg/L})(9.81\,\text{m/s}^2)(211\,\text{m})\left(365.24\,\text{d}\times\frac{24\ \text{h}}{\text{d}}\right)} = \boxed{2.4\times10^{5}\ \text{L/s}}$$

23 • A spring has a force constant of $1.0\times10^{4}\text{N/m}$. How far must the spring be stretched for its potential energy to equal (*a*) 50 J, and (*b*) 100 J?

Picture the Problem The potential energy of a stretched or compressed ideal spring U_s is related to its force (stiffness) constant k and stretch or compression Δx by $U_s = \frac{1}{2}kx^2$.

(*a*) Relate the potential energy stored in the spring to the distance it has been stretched:

$$U_s = \tfrac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{2U_s}{k}}$$

Substitute numerical values and evaluate x:

$$\boldsymbol{x} = \sqrt{\frac{2(50\,\text{J})}{1.0\times10^{4}\ \text{N/m}}} = \boxed{10\,\text{cm}}$$

(*b*) Proceed as in (*a*) with U_s = 100 J:

$$\boldsymbol{x} = \sqrt{\frac{2(100\,\text{J})}{1.0\times10^{4}\ \text{N/m}}} = \boxed{14\,\text{cm}}$$

34 •• An Atwood's machine (Figure 7-39) consists of masses m_1 and m_2, and a pulley of negligible mass and friction. Starting from rest, the speed of the two masses is 4.0 m/s at the end of 3.0 s. At that time, the kinetic energy of the system is 80 J and each mass has moved a distance of 6.0 m. Determine the values of m_1 and m_2.

Picture the Problem In a simple Atwood's machine, the only effect of the pulley is to connect the motions of the two objects on either side of it; that is, it could be replaced by a piece of polished pipe. We can relate the kinetic energy of the rising and falling objects to the mass of the system and to their common speed and relate their accelerations to the sum and difference of their masses … leading to simultaneous equations in m_1 and m_2.

Relate the kinetic energy of the system to the total mass being accelerated:

$$K = \tfrac{1}{2}(m_1 + m_2)v^2 \Rightarrow m_1 + m_2 = \frac{2K}{v^2}$$

Substitute numerical values and evaluate $m_1 + m_2$:

$$m_1 + m_2 = \frac{2(80\,\text{J})}{(4.0\,\text{m/s})^2} = 10.0\,\text{kg} \qquad (1)$$

In Chapter 4, the acceleration of the masses was shown to be:

$$a = \frac{m_1 - m_2}{m_1 + m_2}g$$

Because $v(t) = at$, we can eliminate a in the previous equation to obtain:

$$v(t) = \frac{m_1 - m_2}{m_1 + m_2}gt$$

Solving for $m_1 - m_2$ yields:

$$m_1 - m_2 = \frac{(m_1 + m_2)v(t)}{gt}$$

Substitute numerical values and evaluate $m_1 - m_2$:

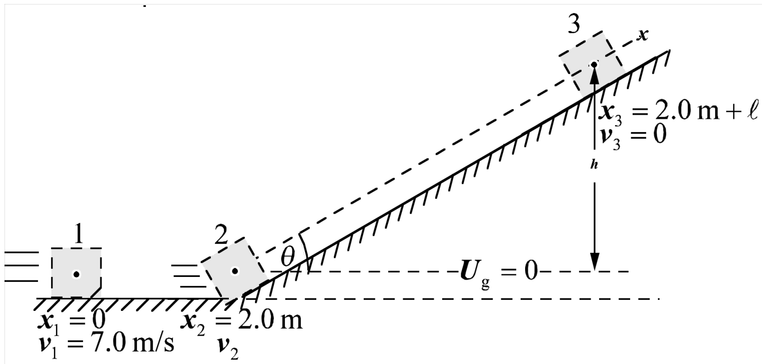
$$m_1 - m_2 = \frac{(10\,\text{kg})(4.0\,\text{m/s})}{(9.81\,\text{m/s}^2)(3.0\,\text{s})} = 1.36\,\text{kg} \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$\boldsymbol{m}_1 = \boxed{5.7\,\text{kg}} \text{ and } \boldsymbol{m}_2 = \boxed{4.3\,\text{kg}}$$

38 • A 3.0-kg block slides along a frictionless horizontal surface with a speed of 7.0 m/s (Figure 7-41). After sliding a distance of 2.0 m, the block makes a smooth transition to a frictionless ramp inclined at an angle of 40° to the horizontal. What distance along the ramp does the block slide before coming momentarily to rest?

Picture the Problem The pictorial representation shows the block in its initial, intermediate, and final states. It also shows a choice for $U_g = 0$. Let the system consist of the block, ramp, and the earth. Because the surfaces are frictionless, the initial kinetic energy of the system is equal to its final gravitational potential energy when the block has come to rest on the incline.



Apply conservation of mechanical energy to the system to obtain:

$$W_{\text{ext}} = ÄK + ÄU$$

or, because $W_{\text{ext}} = 0$,

$$ÄK + ÄU = 0$$

Because $K_3 = U_1 = 0$:

$$-K_1 + U_3 = 0$$

Substituting for K_1 and U_3 yields:

$$-\tfrac{1}{2}mv_1^2 + mgh = 0 \Rightarrow h = \frac{v_1^2}{2g}$$

where h is the change in elevation of the block as it slides to a momentary stop on the ramp.

Relate the height h to the displacement ℓ of the block along the ramp and the angle the ramp makes with the horizontal:

$$h = \ell \sin\theta$$

Equate the two expressions for h and solve for ℓ to obtain:

$$\ell \sin\theta = \frac{v_1^2}{2g} \Rightarrow \ell = \frac{v_1^2}{2g\sin\theta}$$

Substitute numerical values and evaluate ℓ:

$$\ell = \frac{(7.0\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)\sin 40°} = \boxed{3.9\ \text{m}}$$

39 • The 3.00-kg object in Figure 7-42 is released from rest at a height of 5.00 m on a curved frictionless ramp. At the foot of the ramp is a spring of force constant 400 N/m. The object slides down the ramp and into the spring, compressing it a distance *x* before coming momentarily to rest. (*a*) Find *x*. (*b*) Describe the motion object (if any) after the block momentarily comes to rest?

Picture the Problem Let the system consist of the earth, the block, and the spring. With this choice there are no external forces doing work to change the energy of the system. Let $U_g = 0$ at the elevation of the spring. Then the initial gravitational potential energy of the 3.00-kg object is transformed into kinetic energy as it slides down the ramp and then, as it compresses the spring, into potential energy stored in the spring.

(*a*) Apply conservation of mechanical energy to the system to relate the distance the spring is compressed to the initial potential energy of the block:

$$W_{ext} = \Delta K + \Delta U = 0$$

and, because $\Delta K = 0$,

$$-mgh + \tfrac{1}{2}kx^2 = 0 \Rightarrow x = \sqrt{\frac{2mgh}{k}}$$

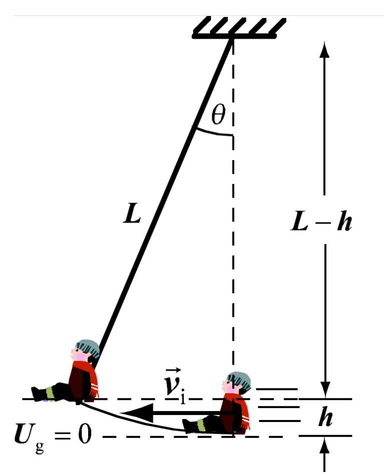
Substitute numerical values and evaluate *x*:

$$x = \sqrt{\frac{2(3.00\,\text{kg})(9.81\,\text{m/s}^2)(5.00\,\text{m})}{400\,\text{N/m}}} = \boxed{0.858\,\text{m}}$$

(*b*) The energy stored in the compressed spring will accelerate the block, launching it back up the incline and the block will retrace its path, rising to a height of 5.00 m.

41 • **[SSM]** A 16-kg child on a 6.0-m-long playground swing moves with a speed of 3.4 m/s when the swing seat passes through its lowest point. What is the angle that the swing makes with the vertical when the swing is at its highest point? Assume that the effects due to air resistance are negligible, and assume that the child is not pumping the swing.

Picture the Problem Let the system consist of the earth and the child. Then $W_{ext} = 0$. Choose $U_g = 0$ at the child's lowest point as shown in the diagram to the right. Then the child's initial energy is entirely kinetic and its energy when it is at its highest point is entirely gravitational potential. We can determine *h* from conservation of mechanical energy and then use trigonometry to determine *θ*.



Using the diagram, relate *θ* to *h* and *L*:

$$\theta = \cos^{-1}\left(\frac{L-h}{L}\right) = \cos^{-1}\left(1 - \frac{h}{L}\right) \quad (1)$$

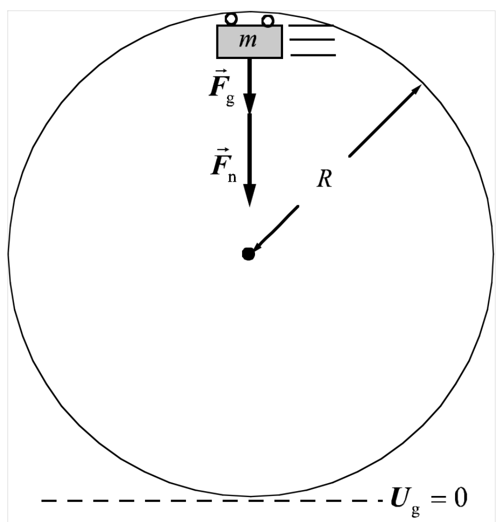
Apply conservation of mechanical

$$W_{ext} = \Delta K + \Delta U = 0$$

energy to the system to obtain:	or, because $K_f = U_{g,i} = 0$, $-K_i + U_{g,f} = 0$
Substituting for K_i and $U_{g,f}$ yields:	$-\frac{1}{2}mv_i^2 + mgh = 0 \Rightarrow h = \frac{v_i^2}{2g}$
Substitute for h in equation (1) to obtain:	$\theta = \cos^{-1}\left(1 - \frac{v_i^2}{2gL}\right)$
Substitute numerical values and evaluate θ:	$\theta = \cos^{-1}\left(1 - \frac{(3.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(6.0\,\text{m})}\right) = \boxed{26°}$

47 •• A 1500-kg roller coaster car starts from rest a height $H = 23.0$ m (Figure 7-46) above the bottom of a 15.0-m-diameter loop. If friction is negligible, determine the downward force of the rails on the car when the upside-down car is at the top of the loop.

Picture the Problem Let the system include the car, the track, and the earth. The pictorial representation shows the forces acting on the car when it is upside down at the top of the loop. Choose $U_g = 0$ at the bottom of the loop. We can express F_n in terms of v and R by apply Newton's 2nd law to the car and then obtain a second expression in these same variables by applying conservation of mechanical energy to the system. The simultaneous solution of these equations will yield an expression for F_n in terms of known quantities.



Apply $\sum F_{radial} = ma_{radial}$ to the car at the top of the circle and solve for F_n:	$F_n + mg = m\frac{v^2}{R}$ and $F_n = m\frac{v^2}{R} - mg$ (1)
Using conservation of mechanical	$W_{ext} = \Delta K + \Delta U = 0$

energy, relate the energy of the car at the beginning of its motion to its energy when it is at the top of the loop:

or, because $K_i = 0$,
$K_f + U_f - U_i = 0$

Substitute for K_f, U_f, and U_i to obtain:

$$\tfrac{1}{2}mv^2 + mg(2R) - mgH = 0$$

Solving for $m\frac{v^2}{R}$ yields:

$$m\frac{v^2}{R} = 2mg\left(\frac{H}{R} - 2\right) \qquad (2)$$

Substitute equation (2) in equation (1) to obtain:

$$F_n = 2mg\left(\frac{H}{R} - 2\right) - mg = mg\left(\frac{2H}{R} - 5\right)$$

Substitute numerical values and evaluate F_n:

$$\boldsymbol{F}_n = (1500\,\text{kg})(9.81\,\text{m/s}^2)\left[\frac{2(23.0\,\text{m})}{7.50\,\text{m}} - 5\right] = \boxed{16.7\,\text{kN}}$$

71 • (*a*) Calculate the rest energy of 1.0 g of dirt. (*b*) If you could convert this energy completely into electrical energy and sell it for $0.10/kW·h, how much money would you take in? (*c*) If you could power a 100-W light bulb with this energy, for how long could you keep the bulb lit?

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

(*a*) The rest energy of the dirt is given by:

$$E_0 = mc^2$$

Substitute numerical values and evaluate E_0:

$$E_0 = (1.0\times10^{-3}\,\text{kg})(2.998\times10^{8}\,\text{m/s})^2 = 8.988\times10^{13}\,\text{J} = \boxed{9.0\times10^{13}\,\text{J}}$$

(*b*) Express kW·h in joules:

$$1\,\text{kW}\cdot\text{h} = (1\times10^{3}\,\text{J/s})\left(1\,\text{h}\times\frac{3600\,\text{s}}{\text{h}}\right) = 3.60\times10^{6}\,\text{J}$$

Convert 8.988×10^{13} J to kW·h:

$$8.988\times10^{13}\text{ J} = \left(8.988\times10^{13}\text{ J}\right)\times\left(\frac{1\text{ kW}\cdot\text{h}}{3.60\times10^{6}\text{ J}}\right) = 2.50\times10^{7}\text{ kW}\cdot\text{h}$$

Determine the price of the electrical energy:

$$\text{Price} = \left(2.50\times10^{7}\text{ kW}\cdot\text{h}\right)\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right) = \boxed{\$2.5\times10^{6}}$$

(c) Relate the energy consumed to its rate of consumption and the time:

$$\Delta E = P\Delta t \Rightarrow \Delta t = \frac{\Delta E}{P}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{8.988\times10^{13}\text{ J}}{100\text{ W}} = 8.988\times10^{11}\text{ s} = \boxed{9.0\times10^{11}\text{ s}}$$

$$= 8.988\times10^{11}\text{ s}\times\frac{1\text{ y}}{3.156\times10^{7}\text{ s}} = \boxed{2.8\times10^{4}\text{ y}}$$

79 •• A large nuclear power plant produces 1000 MW of electrical power by nuclear fission. (a) By how many kilograms does the mass of the nuclear fuel decrease by in one year? (Assume an efficiency of 33 percent for a nuclear power plant.) (b) In a coal-burning power plant, each kilogram of coal releases 31 MJ of thermal energy when burned. How many kilograms of coal are needed each year for a 1000-MW coal-burning power plant? (Assume an efficiency of 38 percent for a coal-burning power plant.)

Picture the Problem The annual consumption of matter by the fission plant is the ratio of its annual energy output to the square of the speed of light. The annual consumption of coal in a coal-burning power plant is the ratio of its annual energy output to energy per unit mass of the coal.

(a) The yearly consumption of matter is given by:

$$\Delta m = \frac{E}{\varepsilon c^2}$$

where E is the energy to be generated and ε is the efficiency of the plant.

Because the energy to be generated is the product of the power output of the plant and the elapsed time:

$$\Delta m = \frac{P\Delta t}{\varepsilon\, c^2} \qquad (1)$$

Substitute numerical values and evaluate Δm:

$$\Delta m = \frac{(1000\text{ MW})\left(1\text{ y}\times\dfrac{3.156\times10^7\text{ s}}{\text{y}}\right)}{(0.33)\,(2.998\times10^8\text{ m/s})^2} = \boxed{1.1\text{ kg}}$$

(*b*) For a coal-burning power plant, equation (1) becomes:

$$\Delta m_{\text{coal}} = \frac{P\Delta t}{\varepsilon\left(\dfrac{\text{energy released}}{\text{unit mass}}\right)}$$

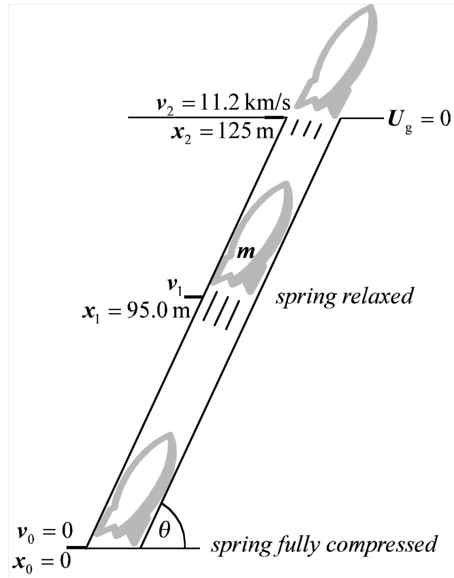
Substitute numerical values and evaluate Δm_{coal}:

$$\Delta m_{\text{coal}} = \frac{(1000\text{ MW})\left(1\text{ y}\times\dfrac{3.156\times10^7\text{ s}}{\text{y}}\right)}{(0.38)\,(31\text{ MJ/kg})} = \boxed{2.7\times10^9\text{ kg}}$$

Remarks: 2.7×10^9 kg is approximately 3 million tons!

92 •• In old science fiction movies, writers attempted to come up with novel ways of launching spacecraft toward the moon. In one hypothetical case, a screenwriter envisioned launching a moon probe from a deep, smooth tunnel, inclined at 65.0° above the horizontal. At the bottom of the tunnel a very stiff spring designed to launch the craft was anchored. The top of the spring, when the spring is unstressed, is 30.0 m from the upper end of the table. The screenwriter knew from his research that to reach the moon, the 318-kg probe should have a speed of at least 11.2 km/s when it exits the tunnel. If the spring is compressed by 95.0 m just before launch, what is the minimum value for its force constant to achieve a successful launch? Neglect friction with the tunnel walls and floor.

Picture the Problem Let the system consist of the earth, spring, tunnel, and the spacecraft and the zero of gravitational potential energy be at the surface of the earth. Then there are no external forces to do work on the system and W_{ext} = 0. We can use conservation of mechanical energy to find the minimum value of the force constant that will result in a successful launch. The pictorial representation summarizes the details of the launch. Note that the spacecraft slows somewhat over the last 30 m of its launch.



(*a*) Apply conservation of mechanical energy to the spacecraft as it moves from $x = x_0$ to $x = x_2$ to obtain:

$$\boldsymbol{W}_{\text{ext}} = \text{Ä}\boldsymbol{E}_{\text{mech}}$$

or, because $W_{ext} = 0$,

$$\Delta E_{\text{mech}} = 0 \qquad (1)$$

The change in the mechanical energy of the system is:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U_g + \Delta U_s = K_2 - K_0 + U_{g,2} - U_{g,0} + U_{s,2} - U_{s,0}$$

Because $K_0 = U_{g,2} = U_{s,2} = 0$:

$$\Delta E_{\text{mech}} = K_2 - U_{g,0} - U_{s,0}$$

Substituting for K_2, $U_{g,0}$, and $U_{s,0}$ yields:

$$\Delta E_{\text{mech}} = \tfrac{1}{2}mv_2^2 - \left(-mgx_2\sin\theta\right) - \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgx_2\sin\theta - \tfrac{1}{2}kx_1^2$$

Substituting for ΔE_{mech} in equation (1) yields:

$$\tfrac{1}{2}\boldsymbol{m}\boldsymbol{v}_2^2 + \boldsymbol{m}\boldsymbol{g}\boldsymbol{x}_2\sin\boldsymbol{\theta} - \tfrac{1}{2}\boldsymbol{k}\boldsymbol{x}_1^2 = 0$$

Solving for k yields:

$$\boldsymbol{k} = \frac{\boldsymbol{m}\boldsymbol{v}_2^2 + 2\boldsymbol{m}\boldsymbol{g}\boldsymbol{x}_2\sin\boldsymbol{\theta}}{\boldsymbol{x}_1^2}$$

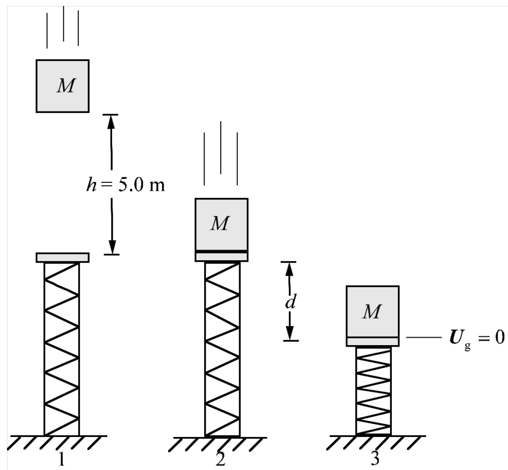
Substitute numerical values and evaluate k:

$$\boldsymbol{k} = \frac{(318\text{ kg})(11.2\text{ km/s})^2 + 2(318\text{ kg})(9.81\text{ m/s}^2)(125\text{ m})\sin 65.0°}{(95.0\text{ m})^2}$$

$$= \boxed{4.42\times10^3\text{ kN/m}}$$

98 •• The cable of a 2000-kg elevator has broken, and the elevator is moving downward at a steady speed of 1.5 m/s. A safety braking system that works on friction prevents the downward speed from increasing. (*a*) At what rate is the braking system converting mechanical energy to thermal energy? (*b*) While the elevator is moving downward at 1.5 m/s, the braking system fails and the elevator is in free-fall for a distance of 5.0 m before hitting the top of a large safety spring with force constant of 1.5×10^4 N/m. After the elevator hits the top of the spring, find the distance *d* that the spring is compressed before the elevator is brought to rest.

Picture the Problem The rate of conversion of mechanical energy can be determined from $P = \vec{F} \cdot \vec{v}$. The pictorial representation shows the elevator moving downward just as it goes into freefall as state 1. In state 2 the elevator is moving faster and is about to strike the relaxed spring. The momentarily at rest elevator on the compressed spring is shown as state 3. Let $U_g = 0$ where the spring has its maximum compression and the system consist of the earth, the elevator, and the spring. Then $W_{ext} = 0$ and we can apply conservation of mechanical energy to the analysis of the falling elevator and compressing spring.



(*a*) Express the rate of conversion of mechanical energy to thermal energy as a function of the speed of the elevator and braking force acting on it:

$$P = F_{braking} v_0$$

Because the elevator is moving with constant speed, the net force acting on it is zero and:

$$F_{braking} = Mg$$

Substitute for $F_{braking}$ to obtain:

$$P = Mgv_0$$

Substitute numerical values and evaluate *P*:

$$P = (2000\,\text{kg})(9.81\,\text{m/s}^2)(1.5\,\text{m/s}) = \boxed{29\,\text{kW}}$$

(*b*) Apply the conservation of mechanical energy to the falling elevator and compressing spring:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_3 = U_{g,3} = U_{s,1} = 0$:

$$-\tfrac{1}{2}\boldsymbol{M}\boldsymbol{v}_0^2 - \boldsymbol{Mg}(\boldsymbol{h} + \boldsymbol{d}) + \tfrac{1}{2}\boldsymbol{kd}^2 = 0$$

Rewrite this equation as a quadratic equation in *d*, the maximum compression of the spring:

$$\boldsymbol{d}^2 - \left(\frac{2\boldsymbol{Mg}}{\boldsymbol{k}}\right)\boldsymbol{d} - \frac{\boldsymbol{M}}{\boldsymbol{k}}\left(2\boldsymbol{gh} + \boldsymbol{v}_0^2\right) = 0$$

Solve for *d* to obtain:

$$d = \frac{Mg}{k} \pm \sqrt{\frac{M^2g^2}{k^2} + \frac{M}{k}\left(2gh + v_0^2\right)}$$

Substitute numerical values and evaluate *d*:

$$\boldsymbol{d} = \frac{(2000\,\text{kg})(9.81\,\text{m/s}^2)}{1.5\times10^4\ \text{N/m}} + \sqrt{\frac{(2000\,\text{kg})^2(9.81\,\text{m/s}^2)^2}{(1.5\times10^4\ \text{N/m})^2} + \frac{2000\,\text{kg}}{1.5\times10^4\ \text{N/m}}\left[2(9.81\,\text{m/s}^2)(5.0\,\text{m}) + (1.5\,\text{m/s})^2\right]}$$

$$= \boxed{5.2\,\text{m}}$$

107 ••• A rock climber is rappelling down the face of a cliff when his hold slips and he slides down over the rock face, supported only by the bungee cord he attached to the top of the cliff. The cliff face is in the form of a smooth quarter-cylinder with height (and radius) $H = 300$ m (Figure 7-58). Treat the bungee cord as a spring with force constant $k = 5.00$ N/m and unstressed length $L = 60.0$ m. The climber's mass is 85.0 kg. (*a*) Using a **spreadsheet** program, make a graph of the rock climber's potential energy as a function of *s*, his distance from the top of the cliff *measured along the curved surface*. Use values of *s* between 60.0 m and 200 m. (*b*) His fall began when he was a distance $s_i = 60.0$ m from the top of the cliff, and ended when he was a distance $s_f = 110$ m from the top. Determine how much energy is dissipated by friction between the time he initially slipped and the time when he came to a stop.

Picture the Problem The potential energy of the climber is the sum of his gravitational potential energy and the potential energy stored in the spring-like bungee cord. Let θ be the angle which the position of the rock climber on the cliff face makes with a vertical axis and choose the zero of gravitational potential energy to be at the bottom of the cliff. We can use the definitions of U_g and U_{spring} to express the climber's total potential energy and the work-energy theorem for problems with friction to determine how energy is dissipated by friction between the time he initially slipped and finally came to a stop.

(*a*) The total potential energy of the climber is the sum of $U_{\text{bungee cord}}$ and U_g:

$$U(s) = U_{\text{bungee cord}} + U_g \qquad (1)$$

$U_{\text{bungee cord}}$ is given by:

$$U_{\text{bungee cord}} = \tfrac{1}{2}k(s-L)^2$$

U_g is given by:

$$U_g = Mgy = MgH\cos\theta = MgH\cos\left(\frac{s}{H}\right)$$

Substitute for $U_{\text{bunge cord}}$ and U_g in equation (1) to obtain:

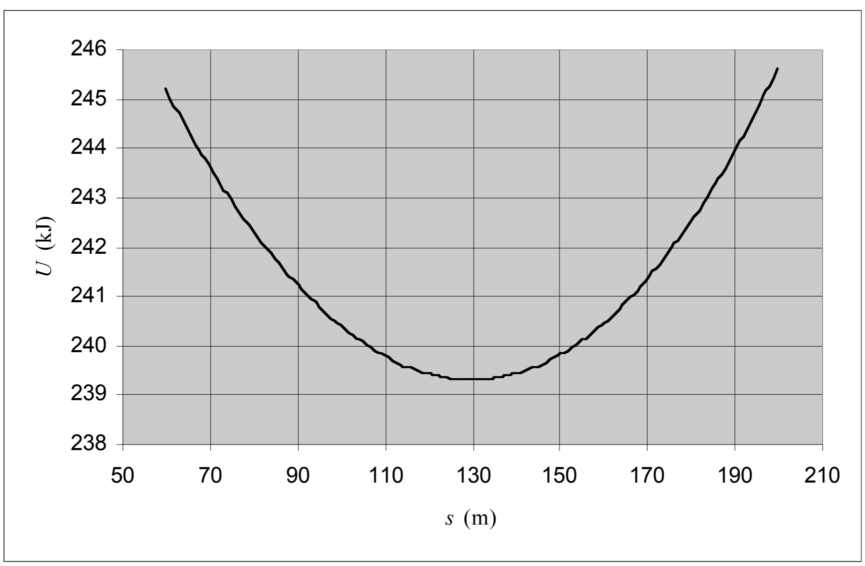
$$U(s) = \tfrac{1}{2}k(s-L)^2 + MgH\cos\left(\frac{s}{H}\right)$$

A spreadsheet solution is shown below. The constants used in the potential energy function and the formulas used to calculate the potential energy are as follows:

Cell	Content/Formula	Algebraic Form
B3	300	H
B4	5.00	k
B5	60.0	L
B6	85.0	M
B7	9.81	g
D11	60.0	s
D12	D11+1	$s+1$
E11	0.5*$B$4*(D11−$B$5)^2 +$B$6*$B$7*$B$3*(cos(D11/$B$3))	$\frac{1}{2}k(s-L)^2 + MgH\cos\left(\frac{s}{H}\right)$

	A	B	C	D	E
1					
2					
3	H =	300	m		
4	k =	5.00	N/m		
5	L =	60.0	m		
6	m =	85.0	kg		
7	g =	9.81	m/s^2		
8					
9				s	$U(s)$
10				(m)	(J)
11				60	2.452E+05
12				61	2.450E+05
13				62	2.448E+05
14				63	2.447E+05
15				64	2.445E+05
59				108	2.399E+05
60				109	2.398E+05
61				110	2.398E+05
62				111	2.397E+05
63				112	2.397E+05

The following graph was plotted using the data from columns D (s) and E ($U(s)$).



(b) Apply the work-kinetic energy theorem for problems with friction to the climber to obtain:

$$W_{ext} = \Delta K + \Delta U(s) + XE_{therm}$$

or, because $W_{ext} = \Delta K = 0$,

$$\Delta U(s) + \Delta E_{therm} = 0$$

Solve for the energy dissipated by friction to obtain:

$$\Delta E_{\text{therm}} = -\Delta U(s) = -\left(U(110\text{ m}) - U(60.0\text{ m})\right) = -U(110\text{ m}) + U(60.0\text{ m})$$

Substituting for U(110 m) and U(60.0 m) and simplifying yields:

$$\begin{aligned}\Delta E_{\text{therm}} &= -\left[\tfrac{1}{2}k(110\text{ m} - L)^2 + MgH\cos\left(\frac{110\text{ m}}{H}\right)\right] \\ &\quad + \left[\tfrac{1}{2}k(60.0\text{ m} - L)^2 + MgH\cos\left(\frac{60.0\text{ m}}{H}\right)\right] \\ &= -\tfrac{1}{2}k(110\text{ m} - L)^2 - MgH\cos\left(\frac{110\text{ m}}{H}\right) + \tfrac{1}{2}k(60.0\text{ m} - L)^2 \\ &\quad + MgH\cos\left(\frac{60.0\text{ m}}{H}\right)\end{aligned}$$

Because L = 60.0 m, the third term is zero. Simplifying yields:

$$\begin{aligned}\Delta E_{\text{therm}} &= -\tfrac{1}{2}k(110\text{ m} - L)^2 - MgH\cos\left(\frac{110\text{ m}}{H}\right) + MgH\cos\left(\frac{60.0\text{ m}}{H}\right) \\ &= -\tfrac{1}{2}k(110\text{ m} - L)^2 - MgH\left[\cos\left(\frac{110\text{ m}}{H}\right) - \cos\left(\frac{60.0\text{ m}}{H}\right)\right]\end{aligned}$$

Substitute numerical values and evaluate ΔE_{therm}:

$$\begin{aligned}\Delta E_{\text{therm}} &= -\tfrac{1}{2}(5.00\text{ N/m})(110\text{ m} - 60.0\text{ m})^2 \\ &\quad - (85.0\text{ kg})(9.81\text{ m/s}^2)(300\text{ m})\left[\cos\left(\frac{110\text{ m}}{(300\text{ m})}\right) - \cos\left(\frac{60.0\text{ m}}{(300\text{ m})}\right)\right] \\ &\approx \boxed{5.4\text{ kJ}}\end{aligned}$$

Remarks: You can obtain this same result by examining the partial spreadsheet printout or the graph shown above.